Response to "Crime Places in Context"

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The recent paper "Crime Places in Context: An Illustration of the Multilevel Nature of Hot Spot Development" (Deryol et al, 2016), published in the *Journal of Quantitative Criminology*, uses multilevel Poisson regression analysis to evaluate factors which contribute to local crime rates. In particular, the authors test three hypotheses concerning the three-way interaction between nearby carry-out liquor stores, on-premises drinking establishments, and bus routes, to determine whether "it is a combination of risky nodes and paths that are more important than any one single risky facility". However, the analysis contains statistical flaws which render it unable to test the hypotheses and invalidate the conclusions presented. Most importantly, their interaction term is not an interaction at all.

Table 2 of Deryol et al (2016) shows three hierarchical Poisson regression models fit to the data. Model 2 does not contain an interaction term, fitting the three covariates separately, while Model 3 contains an interaction term but no main effects. The stated hypotheses are supported if the interaction model fits the data significantly better, since the interaction represents the joint effect of all three factors. According to the text, the main effects are the log-transformed distances to the nearest liquor stores, drinking establishments, and bus routes, so a simplified linear version of Model 2 would be

$$y = \beta_0 + \beta_1 \log X_1 + \beta_2 \log X_2 + \beta_3 \log X_3, \tag{1}$$

where β_1, β_2, and β_3 are the slope coefficients and X_1, X_2, and X_3 the three distances. (I am ignoring the random effects for clarity.) Model 3 is the interaction model, but the authors state

> Finally, since one of our main hypotheses (Hypothesis 1) was that the effects of these three distance measures were contingent upon one another, a three-way interaction was created by multiplying the original distances and then logging the product term.

Unless this sentence is in error, this suggests that Model 3 has the form

$$y = \beta_0 + \beta_1 \log(X_1 X_2 X_3), \tag{2}$$

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again ignoring the random effects. However, it is a well-known property of logarithms that $\log(ab) = \log(a) + \log(b)$, and so Equation (2) can equivalently be written as

$$y = \beta_0 + \beta_1(\log X_1 + \log X_2 + \log X_3). \tag{3}$$

In other words, Model 3 is *not* an interaction model: it is an additive model like Model 2, equivalent to Equation (1) with $\beta_1 = \beta_2 = \beta_3$. This means the model has no relevance to the stated hypotheses (Hypotheses 1 and 2) about the interaction between the three distances: it is simply Model 2 with the additional assumption that the three distances each have identical effects on crime. It does not measure the joint effect of the distances or whether "the effects of these three distance measures were contingent upon one another".

Given the erroneous analysis, either the evidence presented in the paper cannot support the conclusions drawn or the description of the analysis is erroneous and should be corrected. This criticism also applies to the models presented in Table 3, if the variables are transformed in the same way, and hence undermines the analysis there as well.

The correct approach for interaction terms of transformed variables is to multiply the transformed variables—that is, the correct Model 3 would have been

$$y = \beta_0 + \beta_1 \log(X_1) \log(X_2) \log(X_3).$$

This avoids the problem.

An additional problem appears when the authors switch to linear hierarchical models to test which model in Table 2 fits best. They use deviance tests to compare the three models, finding that the difference between Models 2 and 3 "was not significant… ($p < 0.05$) *[sic]*". This comparison is problematic: deviance tests can only compare nested models, where one model contains a subset of the covariates of the other, and Models 2 and 3 are not nested. Model 3 is both more restrictive (because its "interaction" is equivalent to the three variables in Model 2, but with equal coefficients) and less restrictive, as it contains an additional random effect term. If the interaction term problem is corrected, the models are still not nested, for similar reasons.

The authors should clarify how they performed this test in a statistically sound way when standard deviance tests are not applicable; otherwise, it is not clear how they can justify the choice of one model over the other.

References

Deryol R, Wilcox P, Logan M, Wooldredge J (2016) Crime Places in Context: An Illustration of the Multilevel Nature of Hot Spot Development. Journal of Quantitative Criminology pp 1–21, DOI 10.1007/s10940-015-9278-1